A Mechanistic Model for Two-Phase Bubble Flow in Vertical Pipes

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The flow of gas-liquid mixtures in vertical pipes is widely encountered in the petroleum industry. In most oil wells gas is produced simultaneously and the oil production rate is dependent upon the gas rate. As the gas rate is increased from zero, the gas is dispersed as discrete bubbles that increase steadily in number and, with coalescence and pressure drop, may increase in size. This flow pattern is called bubble flow. At progressively higher gas velocities slug, froth, and annular-mist flow patterns occur.

For a given gas rate the volume fraction of the gas phase or gas void fraction is dependent upon the oil rate. For a naturally producing oil well, the oil production rate depends upon the difference between the flowing bottom hole pressure and the surface pressure. This difference is dependent upon the flow pattern and the holdup, which are dependent upon the oil and gas rates, the fluid properties, the depth and diameter of the well, and the absolute pressure and temperature.

Numerous empirical correlations are available (Brill and Beggs, 1982) for predicting flow pattern, holdup, and pressure drop in oil wells. It has only been in recent years that researchers have begun to consider the basic hydrodynamic phenomena involved in two-phase flow and to develop theoretical models for determining flow pattern, holdup, and pressure drop. Mechanistic models have been developed for flow pattern (Taitel et al., 1980), holdup (Barnea and Brauner, 1985), and pressure drop in slug flow (Fernandes et al., 1983; Sylvester, 1987) and annular-mist flow (Yao and Sylvester, 1987) in vertical pipes.

The purpose of this paper is to formulate and evaluate a mechanistic model for two-phase bubble flow in vertical pipes that permits calculation of liquid holdup and pressure drop.

Flow Pattern Transition Boundaries

The model is formulated based on the assumption that the flow is fully developed and stable. It is further assumed that at any given location the gas is discretely and uniformly dispersed as bubbles in the liquid phase.

When gas and liquid flow concurrently upward at a low gas rate, bubble flow occurs. Taitel et al. (1980) and Barnea et al. (1982) have developed relations that delineate the bubble flow pattern. At low gas and liquid rates bubble flow exists provided the pipe diameter is larger than

$$D > 19.01\left[\frac{(\rho_L-\rho_G)\sigma}{\rho_L^2 g}\right]^{1/2} \tag{1}$$

According to this relation bubble flow does not exist for small-diameter pipes because the small bubbles rise faster than the Taylor bubbles, which causes coalescence.

Taitel et al. considered two distinct bubble flow patterns, bubbly and dispersed bubble flow, depending upon the gas and liquid rates and the gas void fraction. At low liquid and gas rates the transition from bubbly to slug flow is based on a liquid holdup of 0.25. This transition in terms of superficial liquid and gas velocities is given by

$$V_{SG} = \frac{1}{3}(V_{SL} + 0.75\,u_o) \tag{2}$$

where u_o is the slip or bubble rise velocity given by Harmathy (1960)

$$u_o = 1.53\left[\frac{g\sigma(\rho_L-\rho_G)}{\rho_L^2}\right]^{1/4} \tag{3}$$

At higher gas and liquid rates both bubbly and slug flow become dispersed bubble flow. The transition to dispersed bubble flow is given by

$$2\left[\frac{0.4\sigma}{(\rho_L-\rho_G)g}\right]^{1/2}\left(\frac{\rho_L}{\sigma}\right)^{3/5}\left[\frac{2}{D}C_L\left(\frac{D}{\nu_L}\right)^{-n}\right]^{2/5}(V_{SL}+V_{SG})^{2(3-n)/5} = 0.725 + 4.15\left(\frac{V_{SG}}{V_{SL}+V_{SG}}\right)^{0.5} \tag{4}$$

where $C_L = 0.046$ and $n = 0.2$.

This relation was presented by Barnea et al. (1982), who showed it to agree somewhat better with experimental results than the original expression presented by Taitel et al. (1980). Equation 4 is valid for gas void fraction less than 0.52. For higher values the transition is given by

$$V_{SG} = 1.083(V_{SL} + 0.48u_o) \quad (5)$$

The second term of Eq. 5, which accounts for slippage between the phases, is less than 3% of V_{SL} and therefore can be neglected. This permits dispersed bubble flow to be approximated as a homogeneous flow with no slippage between the gas bubbles and the liquid.

Liquid Holdup and Pressure Drop

The pressure drop for two-phase bubble flow in vertical pipes is strongly dependent upon the liquid holdup. For dispersed bubble flow the no-slip liquid holdup can be used. It is defined by

$$H_L = \lambda_L = \frac{V_{SL}}{(V_{SL} + V_{SG})} \quad (6)$$

For bubbly flow, the liquid holdup also depends upon the slippage between the two phases. Following Wallis (1969), Fernandes (1983), and Caetano (1985), an implicit relation for the liquid holdup is obtained as given by

$$(H_L)^{1/2}u_o = \frac{V_{SG}}{(1 - H_L)} - \frac{V_{SL}}{H_L} \quad (7)$$

The total pressure gradient consists of an elevation term, $(dp/dz)_E$, a friction term, $(dp/dz)_f$, and an acceleration term, $(dp/dz)_A$. Neglecting the accelerational pressure drop, the total pressure gradient becomes

$$\frac{dp}{dz} = \rho_{TP}g + \frac{f\rho_{TP}(V_{SG} + V_{SL})^2}{2D} \quad (8)$$

where the two-phase density is given by

$$\rho_{TP} = H_L\rho_L + (1 - H_L)\rho_G \quad (9)$$

and the friction factor, f, is determined from the modified Zigrang-Sylvester (1982) equation

$$\frac{1}{\sqrt{f}} = -2.0\log\left\{\frac{\left(\frac{\epsilon}{D}\right)}{3.7} - \frac{5.02}{Re_{TP}}\log\left[\frac{\left(\frac{\epsilon}{D}\right)}{3.7} + \frac{13}{Re_{TP}}\right]\right\} \quad (10)$$

The two-phase Reynolds number is defined by

$$Re_{TP} = \frac{D\rho_{TP}(V_{SL} + V_{SG})}{\mu_{TP}} \quad (11)$$

where

$$\mu_{TP} = H_L\mu_L + (1 - H_L)\mu_G \quad (12)$$

In Eqs. 9 and 12, H_L is the no-slip holdup for dispersed bubble flow, Eq. 6, and the actual holdup for bubbly flow, Eq. 7.

Table 1. Required Input Data

Well depth, Z	Bottomhole temperature, T_B
Tubing diameter, D	Surface temperature, T_S
Tubing roughness, ϵ	Bottomhole pressure, p_{wf}
Liquid rate, Q_L	Surface pressure, p_S
Gas rate, Q_G	

Table 2. Required Fluid Property Data

Liquid density, ρ_L
Gas density, ρ_G
Liquid viscosity, μ_L
Gas viscosity, μ_G
Interfacial tension, σ

This bubble flow model requires an iterative trial-and-error calculation. A computer program was developed to calculate surface pressure given bottomhole conditions and vice versa. The computer program utilizes a standard pressure traversing technique with a linear temperature profile and the input data listed in Table 1. In addition, the fluid properties shown in Table 2 were determined using a black oil model.

Comparson with Field Data

The effectiveness of the bubble flow model was evaluated by comparison of its predictions with field data. The sources of gas-oil field data are shown in Table 3. Most of the data were obtained from the Tulsa University Fluid Flow Projects (TUFFP) data bank. Only those cases that showed bubble flow for at least 75% of the total depth were considered.

Table 4 shows that the model predicts the measured pressure drop reasonably well, with an average deviation of −2.1%, an absolute average deviation of 3.7%, and a standard deviation of 4.7%. The average deviation of −2% indicates that the model underpredicts the pressure drop. This is due to the fact that bubble flow was assumed to exist over the entire pipe length.

Table 3. Oil-Gas Data Sources

Asheim (1986)
Chierici et al. (1974)
Espanol (1968)
Govier & Fogarasi (1975)
Messulam (1970)
Orkiszewski (1967)
Poettmann & Carpenter (1952)

Table 4. Statistical Comparison of Pressure Drop Predictions with Experimental Oil-Gas Data

	No. of Cases	Avg. Dev. %	Abs. Avg. Dev. %	Std. Dev. %
This work	23	−2.1	3.7	4.7
Beggs & Brill (1973)	23	2.0	7.1	9.3
Duns & Ros (1963)	23	0.3	4.4	5.5
Hagedorn & Brown (1965)	22	−0.6	4.1	5.1
Mukerjee & Brill (1985)	22	−3.4	5.5	6.2
Orkiszewski (1967)	22	1.1	5.0	5.9

Comparison with Empirical Correlations

Five commonly used empirical correlations were utilized to further evaluate the model. The empirical correlations chosen were those of Duns and Ros (1963), Hagedorn and Brown (1965), Beggs and Brill (1973), Orkiszewski (1967), and Mukerjee and Brill (1985).

Table 4 summarizes the statistical comparison of the model with the empirical correlations. The model outperforms each of the correlations, showing a lower absolute average difference and a lower standard deviation.

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Notation

- C_L = lift coefficient
- D = tubing diameter
- f = friction factor
- g = acceleration due to gravity
- H_L = liquid holdup
- n = exponent
- p = pressure
- p_S = surface pressure
- p_{wf} = flowing bottomhole pressure
- Q_L = volumetric flow rate of liquid
- Q_G = volumetric flow rate of gas
- Re_{TP} = two-phase Reynolds number
- T_B = bottomhole temperature
- T_S = surface temperature
- u_o = bubble rise velocity
- V_{SG} = superficial gas velocity
- V_{SL} = superficial liquid velocity
- z = depth increment
- Z = well depth

Greek letters

- ϵ = absolute pipe roughness
- ρ_G = gas density
- ρ_L = liquid density
- ρ_{TP} = two-phase density
- σ = gas-liquid interfacial tension
- μ_G = gas viscosity
- μ_L = liquid viscosity
- μ_{TP} = two-phase viscosity
- λ = no-slip holdup

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